

## FINAL EXAM,

Two Hours, Full Marks – 50

State any result done in the class which you are using.  
If you use any result not done on the class, do give a proof

1. State clearly the effective Landau theorem and deduce Siegel's theorem from it.  
[10 marks]

2. Let  $\chi$  be a quadratic character mod  $q$ . Show that  $L(1, \chi) \ll \log q$ .  
[10 marks]

3. Let

$$\psi(x) = \sum_{n \leq x} \Lambda(n) \quad \text{and} \quad \psi(x, \chi_0) = \sum_{n \leq x} \Lambda(n) \chi_0(n)$$

where  $\chi_0$  be the trivial Dirichlet character mod  $q$ . Assume that

$$\psi(x) = x + O(x \exp(-c\sqrt{\log x})).$$

Show that

$$\psi(x, \chi_0) = x + O(x \exp(-d\sqrt{\log x})).$$

for some other constant  $d$ .

[10 marks]

4. Let  $\chi$  be a primitive complex character. Then show that the following sum over its zeroes with ordinate less than 1 satisfies

$$\sum_{|\gamma| < 1} \frac{1}{\rho} \ll \log^2 q.$$

[20 marks]