

END-SEMESTER EXAMINATION AUGUST-NOVEMBER 2023
ALGEBRAIC NUMBER THEORY

Instructions: The total time allotted for this examination is 3 hours. This is a closed book examination. Books, notebooks, cellphones, laptops and any such objects that may enable you to get external help are not allowed. Be brief but precise in your answers. You should justify your assertions.

(1) Compute the class number of $\mathbb{Q}(\sqrt{-35})$. ?

~~(2)~~ Prove or disprove: An Artinian Dedekind domain is a PID.

~~(3)~~ Show that there exists a quadratic field whose class number is divisible by 64.

~~(4)~~ Prove or disprove: The fields \mathbb{Q}_p , for p a finite prime or ∞ , have no non-trivial automorphisms.

~~(5)~~ If L/\mathbb{Q} is a finite extension totally ramified and tamely ramified at p a prime in \mathbb{Z} , and K is an intermediate field, what are the possibilities for the exponent of p in $\text{Disc}(K/\mathbb{Q})$?

~~(6)~~ Prove or disprove: If $K = \mathbb{Q}(\sqrt{-23})$, then for α denoting a root of the polynomial $x^3 - x + 1$, the extension $K(\alpha)/K$ is unramified everywhere.

(7) **Extra credit:** Show that for each prime p , the equation $3x^4 + 4y^4 - 19z^4 = 0$ has solutions with $x, y, z \in \mathbb{Q}_p$ which are not all zero, but no such solutions with $x, y, z \in \mathbb{Q}$.