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2
Syzygies Jan–Apr 2023
Midterm exam

3 **Instructions**

- 4 1. This is a take-home examination. You are allowed to use results proved in the following books
5 and your class notes, but nothing else: [Eis95], [Eis05], [Mat80], [Mat89], [Ser00]. Please cite
6 appropriately.
7 2. You are not allowed to discuss among yourselves.
8 3. If you know Macaulay2 or Singular, you may use them to help with calculations, if you want to
9 avoid calculations by hand. If you use these computer algebra systems, you must include the code
10 in your submission, and it must work as it is when I run it.
11 4. Considerable textual match in the solution to any question (or part of a question) with the so-
12 lution from another student or any source other than those mentioned above will be treated as
13 plagiarism.
14 5. Violation of the above will be treated as academic misconduct and will result in getting an F in the
15 course.
16 6. If you have questions, please ask me.
17 7. Submit in class on Tuesday 2022-Mar-14.
18 8. Total marks: 75

19 **Questions**

- 20 1. (10 points) [Eis95] Exercise 19.15
21 2. (10 points) [Eis05] Section 2D Exercise 4
22 3. (20 points) [Eis05] Section 2D Exercise 8 (Each part is worth 5 marks.)
23 4. (20 points) Read [Eis05] Section 2C. Show that the Hilbert function of 7 points in linearly general
24 position in \mathbb{P}^3 is indeed the one given in that section. You can look at Exercise 9 of Section 2D for
25 hints.
26 5. (5 points) Explain where the unmixedness theorem [Mat89, Chapter 17] fails for $\mathbb{k}[[u, v, x, y]]/(ux, uy, vx, vy)$.
27 6. (10 points) Let $S = \mathbb{k}[x_1, \dots, x_n]$, with $\deg x_i = 1$ for each i . Let M be a finitely generated graded
28 S -module. Show that $\text{reg } M = \max\{j - i \mid \beta_{i,j} \neq 0, \text{ht Ann } M \leq i \leq \text{pd } M\}$. Hint: Consider
29 $\text{Hom}_S(F_\bullet, S)$ for a minimal free resolution F_\bullet of M .

30 **References**

- 31 [Eis95] D. Eisenbud. *Commutative algebra, with a View Toward Algebraic Geometry*, volume 150 of *Graduate*
32 *Texts in Mathematics*. Springer-Verlag, New York, 1995. 1
33 [Eis05] D. Eisenbud. *The Geometry of Syzygies*, volume 229 of *Graduate Texts in Mathematics*. Springer-
34 Verlag, New York, 2005. A second course in commutative algebra and algebraic geometry. 1
35 [Mat80] H. Matsumura. *Commutative algebra*, volume 56 of *Mathematics Lecture Note Series*. Ben-
36 jamin/Cummings Publishing Co., Inc., Reading, Mass., second edition, 1980. 1

- 37 [Mat89] H. Matsumura. *Commutative ring theory*, volume 8 of *Cambridge Studies in Advanced Mathematics*.
38 Cambridge University Press, Cambridge, second edition, 1989. Translated from the Japanese
39 by M. Reid. [1](#)
- 40 [Ser00] J.-P. Serre. *Local algebra*. Springer Monographs in Mathematics. Springer-Verlag, Berlin, 2000.
41 Translated from the French by CheeWhye Chin and revised by the author. [1](#)