Programming Language Concepts

End-Semester Examination, II Semester 2022-2023

Date : 27 April, 2023
Time : 1400 – 1700

Marks : 40
Weightage : 40%

General instructions:

1. Write legibly. Properly parenthesize your lambda expressions and use spacing to ensure that they are readable.
2. Code fragments should follow Java syntax as reasonably as possible. You can miss a few semicolons, but use the correct keywords and constructs. Otherwise marks will be deducted.
3. You can assume the following standard encodings introduced in the lecture slides, with the appropriate behaviour:

f^0x	:=	x	$f^{i+1}x$	:=	$f(f^ix)$
$[n]$	:=	$\lambda fx \cdot f^n x$	**succ**	:=	$\lambda pfx \cdot f(pfx)$
plus	:=	$\lambda pqfx \cdot pf(qfx)$	**pair**	:=	$\lambda xyw \cdot wxy$
fst	:=	$\lambda p \cdot p$ **true**	**snd**	:=	$\lambda p \cdot p$ **false**
true	:=	$\lambda xy \cdot x$	**false**	:=	$\lambda xy \cdot y$
ite	:=	$\lambda bxy \cdot bxy$	**iszero**	:=	$\lambda x \cdot (x(\lambda z \cdot$ **false**$))$ **true**

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1. A particular lecture room at CMI (with seating capacity ten) is used both for lectures and discussions. Any member of the institute can join or leave a lecture at any time, as long as there is space. If someone wishes to join a lecture and there is no space, they wait for someone to leave and then join the lecture. A lecture is deemed to have begun when the first person successfully "joins" the lecture (when the room is empty), and it is deemed to have ended when the last person leaves the lecture (at which point the room will be empty again).

   A discussion involves only two people, and no new person can join a discussion. When the discussion ends, both people leave the room together. We require that a lecture and a discussion cannot happen at the same time.

   Assume that the members of the institute are numbered 0 to 99. Design the `LectureRoom` class with methods

   ```
   public void joinLecture(int i)
   public void leaveLecture(int i)
   public void startDiscussion(int i, int j)
   public void endDiscussion()
   ```

   The arguments are the identities of the institute members who wish to join or leave a lecture, or start a discussion. Note that a discussion involves two people, so we need them both as arguments for the `startDiscussion` method. There are no arguments needed for `endDiscussion`, since both members currently in the discussion have to leave.

   Ensure that your design satisfies all the synchronization conditions stated above. *(10 marks)*

2. Let K := $\lambda xy \cdot x$ and S := $\lambda xyz \cdot xz(yz)$. Reduce the following expressions to normal form.

   (a) I := SKK. (b) B := S(KS)K. (c) M := SII. *(6 marks)*

3. (a) Define a lambda expression **pred** such that

$$\textbf{pred}\ [0] \xrightarrow{*}_\beta [0] \qquad \textbf{pred}\ [n+1] \xrightarrow{*}_\beta [n], \quad \text{for all } n \in \mathbb{N}$$

Prove that your expression satisfies the above property. (*Hint:* Use **pair**, **fst** and **snd** to define an appropriate step function from pairs to pairs, and iterate it on an appropriate initial pair.) *(4 marks)*

(b) Define a lambda expression **subtract** such that for all $m, n \in \mathbb{N}$,

$$\textbf{subtract}\ [m]\ [n] \xrightarrow{*}_\beta [m-n] \quad \text{if } m > n \qquad \textbf{subtract}\ [m]\ [n] \xrightarrow{*}_\beta [0] \quad \text{if } m \leqslant n$$

Prove that your expression satisfies the above property. *(2 marks)*

(c) Define a lambda expression **leq** such that for all $m, n \in \mathbb{N}$,

$$\textbf{leq}\ [m]\ [n] \xrightarrow{*}_\beta \textbf{true} \quad \text{if } m \leqslant n \qquad \textbf{leq}\ [m]\ [n] \xrightarrow{*}_\beta \textbf{false} \quad \text{if } m > n$$

Prove that your expression satisfies the above property. *(2 marks)*

(d) Define a lambda expression **divides** such that for all $m \in \mathbb{N} \setminus \{0\}$ and $n \in \mathbb{N}$,

$$\textbf{divides}\ [m]\ [n] \xrightarrow{*}_\beta \textbf{true} \quad \text{if } m \mid n \qquad \textbf{divides}\ [m]\ [n] \xrightarrow{*}_\beta \textbf{false} \quad \text{if } m \nmid n$$

Prove that your expression satisfies the above property. (*Hint:* You can repeatedly subtract m from n an appropriate number of times, and check if the result is 0. But since **subtract**$[a][b]$ reduces to $[0]$ even if $a < b$, you must only subtract if a certain condition is satisfied. Use the expressions defined in the previous items and the expressions specified in the instructions to construct an appropriate step function. If you do this properly, you can apply this step function n times on $[n]$ and the result will be $[0]$ only if m divides n.) *(6 marks)*

4. Assume that we have the following syntax for types:

$$\sigma, \tau := \textbf{int} \mid \textbf{bool} \mid \sigma \times \tau \mid \sigma \to \tau,$$

where **int** and **bool** are atomic types, and $\sigma \times \tau$ denotes the pair type.

Assume that we have a constant **fix** of type $(\sigma \to \sigma) \to \sigma$ (for any type σ) and the following reduction rule:

$$\textbf{fix}\ F \quad \xrightarrow{*}_\beta \quad F\ (\textbf{fix}\ F).$$

Also assume the following constants with the usual reduction rules, and types specified below:

0	::	**int**	, 1 :: int
pred	::	$\textbf{int} \to \textbf{int}$	
plus	::	$\textbf{int} \to \textbf{int} \to \textbf{int}$	
iszero	::	$\textbf{int} \to \textbf{bool}$	
ite	::	$\textbf{bool} \to \sigma \to \sigma \to \sigma$	for any type σ
pair	::	$\sigma \to \tau \to \sigma \times \tau$	for any types σ, τ
fst	::	$\sigma \times \tau \to \sigma$	for any types σ, τ
snd	::	$\sigma \times \tau \to \tau$	for any types σ, τ

What is the type of the following expression F?

$$F \quad := \quad \lambda f n \cdot \{\ \textbf{ite}\ (\textbf{iszero}\ n)\ (\textbf{pair}\ 0\ 1)\ (\textbf{step}\ (f\ (\textbf{pred}\ n)))\ \},$$

where $\textbf{step} := \lambda p \cdot \textbf{pair}\ (\textbf{snd}\ p)\ (\textbf{plus}\ (\textbf{fst}\ p)\ (\textbf{snd}\ p))$.

If we define $\textbf{fib} := \lambda n \cdot \textbf{fst}(\textbf{fix}\ F\ n)$, show that **fib** 3 reduces to 2. ✓ *(10 marks)*