Theory of Computation
Mid semester exam

Time : 30 Sep 2022, 9:30 am
Duration : 2 hours

Maximum Marks : 35
Weightage : 30%

1. Is $uu^{-1}L = L$? Justify. *(1 marks)*

2. Give an algorithm to check if a given NFA accepts some word of even length. *(3 marks)*

3. For a language L, we define $\mathsf{base}(L) = \{w \in \Sigma^+ \mid w = a_1a_2\dots a_m$ for some $m \in \{1,2,\dots\}, a_i \in \Sigma, \exists n_1,\dots,n_m \in \{1,2,\dots\}, a_1^{n_1}a_2^{n_2}\dots a_m^{n_m} \in L\}$. That is, $\mathsf{base}(L)$ contains words which are obtained from words in L by deleting an arbitrary number of letters, but not everything, from a contiguous block consisting of the same letter. If L is regular then $\mathsf{base}(L)$ is also regular. Describe how to construct an NFA for $\mathsf{base}(L)$, given an NFA for L. *(7 marks)*

4. Consider the language $L = \{a^m + a^n = a^{m+n}\}$ over the alphabet $\Sigma = \{a, +, =\}$
   (a) Is L regular? Justify. *(5 marks)*
   (b) Is L context-free? Justify. *(5 marks)*
   (c) Partition the following set of words according to $\equiv_L$.

$$\{aa + aaa =, aaaa + aa = a, a + aa =, aaa + aaa = aaa, aa + aaa = aa\}$$

(2 marks)

5. Consider the language $L = \{x + y = z \mid x, y, z \in \{0,1\}^*, \mathsf{bin}(x) + \mathsf{bin}(y) = \mathsf{bin}(z)\}$ over the alphabet $\{0, 1, +, =\}$. Here $\mathsf{bin}(x)$ denotes the number denoted by the bitstring x with *most significant bit on the left* notation. For example, $\mathsf{bin}(01101) = 13$.
   Is L context-free? Justify. *(7 marks)*

6. Consider the grammar G below for if-then-else statements.

$$G = (\{S, C\}, \{i, t, e, c, s\}, P, S)$$

   where the set of productions P is:

$$S \to iCtSeS \mid iCtS \mid s$$
$$C \to c$$

   (a) Argue that G is ambiguous. *(2 marks)*
   (b) Can you give an unambiguous grammar for $L(G)$? *(3 marks)*