Theory of Computation

Final Exam, I Semester, 2022–2023

Date : 14 Nov 2022
Duration : 2 hours

Maximum Marks : 40

1. For any language L, let $L' = \{z \mid \exists x, y \; s.t. \; xzz^R y \in L, |x| = |y| = |z|\}$. Is the language L' regular whenever L is? Why/Why not? *(7 marks)*

2. Let L be any language. Let $x \sim_L y$ iff for all z, $zx \in L$ if and only if $zy \in L$. Show that $\sim_L$ is an equivalence relation. Is this relation of finite index whenever L is regular? Why/Why not? Can it be of finite index if L is not regular? Why/Why not? *(7 marks)*

3. Write down a context-free grammar for the language of words over $\{a, b, c\}$ consisting of those words where between every pair of c's the number of a's and b' differ by at most 1. For example *aaacacbcb* is in the language but *abcabbcabbc* is not. *(7 marks)*

4. Is the finiteness problem for CFLs decidable? why/Why not? What about the co-finiteness problem i.e. whether the complement is finite. Why/Why not? *(7 marks)*

5. Consider the language

   $\{\#M \mid M$ halts on infinitely many inputs and fails to halt on infinitely many inputs$\}$

   Is this language recursive/r.e? Why/Why not? *(7 marks)*

6. Consider a two tape finite state machine. This is an automaton $(Q, \Sigma, \delta, s, F)$ whose input consists of a pair of words (x, y). The two words are written on two input tapes and the automaton has heads on both tapes. In each step the automaton can read the next letter on one of the two tapes, move the head on that tape right by one step and change its state. So, $\delta \subseteq (Q \times \{1, 2\} \times \Sigma \times Q)$. A transition of the form $(p, 1, a, q)$ says that from state p one can read a on tape 1 and change state to q (and move the head on tape 1 right by one step).) A pair of inputs is accepted if the automaton enters an accepting state after reading both inputs fully.

   (a) Is the emptiness problem for such automata decidable?

   (b) Is the emptiness of the intersection decidable for such automata? That is given A and B can we determine if there is a pair of words (x, y) that is accepted by both A and B.

   (Either case you need not write out detailed proof. A short clear argument in English should suffice).

   If you have time left you can ponder over, what do the answers to the two questions above tell you about intersection closure for such automata. *(7 marks)*
