Mid Semester examination

B.Sc. II year

Analysis III

Duration: 120 min | Maximum marks: 35

Answer all the questions. First three questions carry **NINE** marks and the last one is for **EIGHT** marks.

Write down the **precise** statements of the results you used in your solution, along with your solution.

Your solutions should be **legible, logical and complete** in order to gain full points.

1. Suppose $\{f_n\}$ is a sequence of differentiable functions whose derivatives are continuous on $[a, b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a, b]$. If $\{f'_n\}$ converges uniformly on $[a, b]$, then show that ~~the~~ $\{f_n\}$ converges uniformly ~~sequence can differentiated term-by-term~~ and if f is the uniform limit of the $\{f_n\}$ then $\{f'_n\}$ converges to f'. Please note that your solution needs to use every condition in the hypothesis.

2. Suppose (X, d) is a compact metric space and T is a map from X to X such that $d(Tx, Ty) = d(x, y)$ for all x and y. Prove that T is ~~not~~

surjective. [Hint: If y is a point in X that is not in the image of T then consider the sequence $y, Ty, T(Ty), ...$]

3. Let f be a continuous real valued function on $\mathbb{R}$ with the property that for each real number x, $f(nx)$ goes to 0 as n tends to ∞. Show that using Baire category theorem, that $f(x)$ goes to 0 as x goes to ∞.

4. State and prove the Lebesgue covering Lemma and use it to prove that a continuous function on a compact metric space is uniformly continuous.