1. Let $\varphi : A \to B$ be a ring hom. and

$\varphi^{\#} : \overset{X}{\operatorname{Spec} B} \to \overset{Y}{\operatorname{Spec} A}$

(a) Show that if φ is surjective, then $\varphi^{\#}$ is a closed embedding.

(b) If φ is injective, then $\varphi^{\#}$ is dominant

i.e; $\overline{\varphi^{\#}(X)} = Y$.

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2. Show that the following sets are canonically isomorphic. Let $\Omega = \overline{\Omega}$, $k \subset \Omega$.

(i) geometric pts of X (an alg. scheme $/ k$)

(ii) $\operatorname{Hom}_{\operatorname{Spec} k}(\operatorname{Spec} \Omega, X)$

(iii) points $x \in X$ plus k-injections

$k(x) \hookrightarrow \Omega$

(iv) $\operatorname{Hom}_{\operatorname{Spec} \Omega}(\operatorname{Spec} \Omega, X_\Omega)$, $X_\Omega = X \times_k \Omega$

(v) closed points of X_Ω.

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3. Define integral morphisms $f : X \to Y$ locally. Show that if $f : X \to Y$ then f is a closed map. Show this by showing:

(a) enough to show $f(X)$ is closed in Y

(b) We may assume X and Y are reduced.

(c) We may assume that f is dominant.

(d) We may assume X, Y affine.

Then conclude.

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4. Show by giving examples:

(a) The underlying top. space of $X \times_S Y$ need not be the fibre product of X and Y over

(b) show that $X \times_k Y$ need not be reduced even if X and Y are reduced.

5. Show that $(X \times_S Y)_{red} = (X_{red} \times_{S_{red}} Y_{red})_{red}$.