CHENNAI MATHEMATICAL INSTITUE

Topology

Quiz-1.

Duration: 1 hour Date: Tuesday, 22nd March, 2024.

- **Answer any THREE questions.**
- **Give brief answers.**

Q1. Suppose that X is a T_1-space. Suppose that $\mathcal{K}$ is a maximal collection of closed subsets of X satisfying the finite intersection property. Show that $\bigcap_{C\in\mathcal{K}} C$ is either empty or is a singleton.

Q2. Show that there is no surjective continuous map $\beta(\mathbb{N}) \to \bar{S}_\Omega = S_\Omega \cup \{\Omega\}$.

Q3. Suppose that Y is any compact metric space. Show that there is a surjective continuous map $\beta(\mathbb{N}) \to Y$.

Q4. Suppose that X is a metric space such that $(x_n)_{n\geq 1}$ is a sequence of distinct points in X such that it has no limit points. Show that there is a continuous function $f : X \to [0, 1]$ such that $f(x_{2n}) = 0, f(x_{2n-1}) = 1$ for all $n \geq 1$. Deduce that , if X is not compact, then $\beta(X)$ is not metrizable.