

CHENNAI MATHEMATICAL INSTITUTE

Topology

Quiz-1.

Duration: 1 hour

Date: Tuesday, 20th Feb, 2024.

- Answer ALL questions.
- Give brief answers.

Q1. Show that the Cantor set  $C \subset I = [0, 1]$  is totally disconnected.

Q2. Find the path component of  $I \times I$  with dictionary order topology, that contains the point  $(0, 0)$ .

Q3. Show that  $S_\Omega$  is not compact.

Q4. Find a subset  $A \subset \mathbb{R}^2$  which is compact, path connected, and such that the closure  $C = \text{cl}(\mathbb{R}^2 \setminus A)$  of the complement of  $A$  is not connected.

Q5. Suppose that  $A$  is an infinite subset of a Hausdorff topological space  $X$  such that  $A$  is a closed and discrete. Show that  $X$  is not compact.