Theory of Computation
Quiz 2

November 2, 2023
Time: 11:50am to 1.00pm
Total marks: 50

Write clear and precise answers.

(1) Recall that a context-free language $L \subseteq \Sigma^*$, for some finite alphabet Σ, is *linear* if it has a context-free grammar $G = (V, \Sigma, P, S)$ with all productions of the form $A \to \alpha B \beta$ or $A \to \alpha$ for variables $A, B \in V$ and terminal strings $\alpha, \beta \in \Sigma^*$.

For a pushdown automaton (PDA) a transition $(p, \gamma) \in \delta(q, a, X)$ is a *pop operation* if $\gamma = \epsilon$. The other transitions are *push operations*.

A *one-turn* pushdown automaton (PDA) is a PDA with the property that, for any input, on any computation path once it does a pop operation it will never use push operations in the rest of the computation.

Show that every linear context-free language can be accepted by a one-turn PDA. **15 marks**

(2) Suppose L is a linear context-free language and R is a regular language. Show that $L \cap R$ is a linear context-free language. **10 marks**

(3) Write the definition of a deterministic PDA. Suppose $L = L(M)$ for a deterministic PDA. Let $L' = \{w \in L \mid \text{no proper prefix of } w \text{ is in } L\}$. Is L' accepted by a deterministic PDA? Justify answer. **10 marks**

(4) Construct a Turing machine M that enumerates $\{0,1\}^*$ in canonical order. Specifically, M will output $x_1 \# x_2 \# \cdots \# x_i \# x_{i+1} \# \cdots$, where the x_i appear in canonical order on the output tape. It will use as subroutine a Turing machine M' that computes x_{i+1} from x_i. Give the transition function of M' in detail, and a high level description for M. **15 marks**