Theory of Computation Assignment 4

Due Date: November 20, 2023

Write clear and concise solutions. It is fine to discuss with others, but your solutions *must* be in your own words that you have fully understood. All problems carry equal marks. Upload solutions on moodle. Note: only handwritten solutions are acceptable.

1. Given as input a pair $\langle M, w \rangle$, where $\langle M \rangle$ is a Turing machine code and $w \in \{0, 1\}^*$ is its input, is the problem of checking if M will halt with blank tape (i.e. each cell of the the tape has the blank symbol 'B') a decidable problem? Justify answer with a proof.

2. Given as input a triple $\langle M, w, N \rangle$, where $\langle M \rangle$ is a Turing machine code $w \in \{0, 1\}^*$ is its input, and N is a positive integer, the problem is to decide if M will ever use more than N tape cells in its computation on w. Is this problem decidable? Justify.

3. To every real number x we associate to it a signed binary expansion $(-1)^{\mathrm{sgn}(x)} b_1 b_2 \ldots b_x \cdot a_1 a_2 a_3 \ldots$, where each $a_i, b_j \in \{0, 1\}$, which is an infinite string over $\{+, -, ., 0, 1\}$. We say that x a *computable real number* if there is a Turing machine M_x that, when started on blank tape will print this string on the output tape, symbol by symbol. Equivalently, M_x takes a positive integer n as input and outputs the n^{th} symbol of this string. Show that the set $\mathbb{R}_T$ of all computable real numbers forms a field containing all the rationals. Does $\mathbb{R}_T$ contain any irrational numbers? Does $\mathbb{R}_T$ contain any transcendental numbers? Give arguments to justify your answers.

4. Solve Problem 8.4 in Hopcroft-Ulman's book on Post-Tag systems.