Theory of Computation Assignment 2

Due Date: October 12, 2023

Write clear and concise solutions. It is fine to discuss with others, but your solutions *must* be in your own words that you have fully understood. All problems carry equal marks. Upload solutions on moodle. Note: only handwritten solutions are acceptable.

1. Let L be some regular language. Prove that $\{w \mid ww \in L\}$ is regular.

2. Suppose M is a DFA with n states and p, q be distinguishable states of M. Let $x \in \Sigma^*$ be the shortest string that distinguishes between p and q. What is a bound on $|x|$ as a function of n? Prove your claim.

3. (Problem 3.29 from Hopcroft-Ullman) In a two-tape 1-way finite automaton, the input is a pair of strings (x, y) with $x \in \Sigma^*$ and $y \in \Gamma^*$, where Σ and Γ could be different alphabets. Each state is designated as reading from tape 1 or tape 2. The input (x, y) is accepted if it reaches the right ends of both x and y in a final state. Give an algorithm to check if the accepted set of pairs $L \subseteq \Sigma^* \times \Gamma^*$ is empty. Give an algorithm to check if it is finite.

4. A shuffle $u \circ v$ of two words $u, v \in \Sigma^*$ is any word of length $|u| + |v|$ that can be split into two subsequences that are u and v. The shuffle $L \circ L'$ of two languages L and L' is the set of all $u \circ v$ such that $u \in L$ and $v \in L'$. Show that $L \circ L'$ is regular if both L and L' are regular. Are contex-free languages also closed under shuffle? Justify.

5. A function $f : \Sigma^* \to \Sigma^*$ is a *reduction* from a language L to a language L' if for all $w \in \Sigma^*$ we have $w \in L$ iff $f(w) \in L'$. Give a Mealy machine that computes a reduction from L to L' where L consists of all $w \in \{0, 1\}^*$ with an odd number of 1's and L' consists of all w with an even number of 1's.

6. Show that the CFG $E \to E + E \mid E * E \mid (E) \mid id$ is an ambiguous grammar. Give an equivalent unambiguous grammar for the language it generates, with a proof that it is unambiguous.

7. Let G be the grammar $S \to aS \mid aSbS \mid \epsilon$. Show that $L(G)$ consists of all strings x over a, b such that every prefix of x has at least as many a's as b's.

8. Show that $L = \{a^i b^{i^2} \mid i \geq 0\}$ is not context-free.

9. Show that $\{ww^R w \mid w \in \{a,b\}^*\}$ is not context-free. Is its complement context-free?

10. Show that $\{ucv^R \mid u, v \in \{a,b\}^*$ and v is not a prefix of $u\}$ is context-free. Is its complement context-free?