Theory of Computation Assignment 1

Due Date: August 31, 2023

Write clear and concise solutions. It is fine to discuss with others, but your solutions *must* be in your own words that you have fully understood. All problems carry equal marks.

1. Let n denote your date of birth as an eight digit number in the MMDDYYYY format. Let $m = n(\text{mod } 64)$ and let $w \in \{0,1\}^*$ be the 6-bit binary representation of m. In case m requires fewer than 6 bits, pad it with leading zeros so that $w \in \{0,1\}^6$. Let $L_w = \{x \in \{0,1\}^* \mid w$ is a substring of $x\}$. Design a DFA that accepts L.

2. Let $\Sigma = \{0,1,2\}$. Any string $w \in \{0,1,2\}^*$ can be treated as the ternary representation of a number $\text{enc}(w)$ (by dropping the leading zeros). Design a DFA that accepts the language $L = \{w \in \Sigma^* \mid \text{enc}(w)$ is divisible by $5\}$.

3. For a language $L \subseteq \Sigma^*$ let $\text{pref}(L) = \{w \in \Sigma^* \mid ww' \in L$ for some $w' \in \Sigma^*\}$. Show that if L is regular then $\text{pref}(L)$ is also regular.

4. Let $\Sigma = \{a_1, a_2, \ldots, a_k\}$. Let L consist of strings $w \in \Sigma^*$ such that the last symbol of w does not occur elsewhere in w. That is, if $w \in L$ then $w = xa$ where $x \in (\Sigma \setminus \{a\})^*$. Give an NFA for L.

5. Let $L = \{0^{k^2} \mid$ for all positive integer $k\}$. Is L^* regular? If so, give a DFA for it with an explanation. Otherwise, prove L^* is not regular.

6. Give a regular language L that has a "small" NFA but any DFA for it is "large". Give an intuitive argument justifying your answer.

7. Let $L \subseteq \{0,1\}^*$ consist of all strings w such that there are two 0's in w separated by a number of positions that is a multiple of 4 (note that 0 is also a multiple of 4). Give a DFA for L.

8. For $w \in \{0,1\}^*$ let $\hat{w}$ denote the reverse of w. E.g. if $w = 011$ then $\hat{w} = 110$. As before, let $\text{enc}(\hat{w})$ denote the integer encoded by $\hat{w}$. Let $L = \{w \in \{0,1\}^* \mid \text{enc}(\hat{w})$ is divisible by $7\}$. Design a DFA for L.

9. For a language $L \subseteq \Sigma^*$ let $\min(L)$ consist of all strings $w \in L$ such that no proper prefix of w is in L. Let $\max(L)$ consist of all strings $w \in L$ such that no proper extension wx is in L. Show that both $\min(L)$ and $\max(L)$ are regular if L is regular.

10. If L is regular then show that $L' = \{ww' \mid w \in L \text{ and } |w'| = k\}$ is also regular.