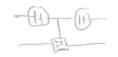
Quantum Computing October 2024, Midsem



SHORT ANSWER TYPE QUESTIONS. EACH QUESTION CARRIES 20 MARKS.

- 1. (a) Prove that the Eigen values of Hermitian matrices are real.
 - (b) Describe how to make classical NAND computation in a reversible way.
 - (c) Describe the circuit and the unitary representation of CNOT gate.
 - (d) Let $|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$ be a quantum state. Measure the first qubit in $\{|+\rangle, |-\rangle\}$ basis. What is the outcome for the second qubit after the measurement.
- (a) Draw the quantum circuit corresponding to the SWAP operation. Describe its unitary operation.
 - (b) Can you construct an unitary U such that $U(|\psi\rangle|\psi\rangle) = |\psi\rangle|0\rangle$ for any qubit state $|\psi\rangle$?
 - (c) Let $Rot_{\theta} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$. Prove that if $Rot_{\theta} \otimes Rot_{\gamma}$ is applied to an EPR pair and measured in standard basis, then the probability of getting the same outcome is $\cos^2 (\theta \gamma)$ and the probability of getting the different outcome is $\sin^2 (\theta \gamma)$.
- 3. (a) Let H be the Hadamard operator on one qubit. Show that

$$H^{\otimes n} = \frac{1}{2^{n/2}} \sum_{x,y \in \{0,1\}^n} (-1)^{x \cdot y} |x\rangle \langle y|.$$

Write an explicit matrix representation of $H^{\otimes 2}$. The trace of a matrix is the sum of the diagonal elements. For any orthonormal basis $\{|i\rangle\}$, prove that $\operatorname{trace}(A) = \sum_i \langle i|A|i\rangle$.

- (b) Construct a CNOT gate from a controlled Z gate and two H gate.
- (c) Draw the circuit for Grover's search algorithm and analyze its complexity.
- (d) Sketch a quantum algorithm that will detect a Hamiltonian cycle in a given graph on n vertices if it exits.
- 4. (a) Construct a quantum circuit that performs the Fourier Transform:

$$|j\rangle \rightarrow \frac{1}{\sqrt{2^n}} \sum_{k=0}^{2^n-1} \omega^{jk} |k\rangle$$

where ω is the complex 2^n -th root of unity. Analyze the complexity of the circuit.

(b) Let

$$|\psi\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle.$$

Show that the operator $2|\psi\rangle\langle\psi|-I$ applied to $\sum_k \alpha_k|k\rangle$ produces

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$$\sum_{k} (-\alpha_k + 2\langle \alpha \rangle) \cdot |k\rangle$$

where $\langle \alpha \rangle = \sum_k \alpha_k / N$.