

Quantum Computing
November 2024, Endsem

SHORT ANSWER TYPE QUESTIONS. EACH QUESTION CARRIES 16 MARKS.

1. Prove that any bounded error query algorithm computing PARITY function over n bits will make $\geq n/2$ queries.
2. Let Alice does the following experiment. She tosses a coin (which is unbiased) and if head comes, she selects the state $|0\rangle$ and else she selects $\cos \theta |0\rangle + \sin \theta |1\rangle$. What is the density matrix for Alice? Compute its Eigen values. What is the Holevo bound corresponding to this? For which value of θ , it is maximum?
3. Use Holevo bound to argue that n qubits can not be used to transmit more than n classical bits.
4. Suppose x_1, x_2, \dots, x_N is a database. Can you design a quantum algorithm that will search the minimum in the database in time $O(\sqrt{N} \log N)$?
5. Let g_1, \dots, g_ℓ and h_1, \dots, h_k are the members of a finite group G presented as a black-box. Can you design a short certificate (which could be a mix of classical and quantum) that shows the subgroup $\langle h_1, \dots, h_k \rangle$ is a proper subgroup of $\langle g_1, \dots, g_\ell \rangle$?

(1928)