

Each question carries 10 marks. justify your steps.
You can use results proved in class, BUT
quote precisely any theorem/formula that you use.
Solution to any problem from HA, if used, should
be provided, (even if we discussed it in class).

- (a) I toss a coin – whose chance of heads is p – till three heads appear and stop. Given that the third head appeared on 50th toss, find the conditional distribution of the time of appearance of second head.
(b) Consider the Bose-Einstein experiment with 20 balls and 10 boxes. Find the most probable number of balls in box 5.
- (a) Let X_1, \dots, X_n be independent random variables each with expectation μ and variance σ^2 . Denote $\bar{X} = \frac{X_1 + \dots + X_n}{n}$. Show that

$$E \left[\frac{1}{n-1} \sum_1^n (X_i - \bar{X})^2 \right] = \sigma^2$$

- (b) We sample, with replacement, from a lot of items in which exactly half are good. How large should be the sample size so that the proportion of good items in the sample is between 0.4 and 0.6 with at least 90% chance.
- Roll a fair die (independently) until each face has appeared, then stop. Let T be the number of rolls needed. *Justifying all your statements*, calculate $E(T)$ and $Var(T)$.
- In a sequence of coin tosses, where chance of Heads of the coin is p ($0 < p < 1$), let X be the length of the run started by the first trial and let Y be the length of the second run. Find joint distribution of X, Y . Are they independent? Calculate $E(X)$ and $E(Y)$.

GOOD LUCK