CMI
BVR

Probability BI
Semestral

26-04-2023
09:30-12:00

Each question carries 12 marks. justify your steps.
You can use results proved in class, BUT
quote precisely any theorem/formula that you use.
Solution to any problem from HA, if used, should
be provided, (even if we discussed it in class).
Relax and Think.

1. (a). From among all subsets of $S = \{1, 2, \ldots, 100\}$ two are selected at random **independently**. Find the probability that the union of the two selected sets equals S.

   (b). $P(A|B_i) = 0.1$ for $1 \leq i \leq 6$. Further, $(B_i : 1 \leq i \leq 6)$ are disjoint events. Calculate $P(A| \overset{6}{\underset{1}{\cup}} B_i)$.

2. (a). A rat is stuck in a maze with three doors. Door 1 leads it to safety after 5 minutes of travel. Door 2 leads it back to the maze after 3 minutes of travel. Door 3 leads it back to the maze after 4 minutes of travel. When in the maze, the rat immediately chooses one of the doors at random. Find the expected time to safety for the rat.

   (b). I roll a fair die. If I get odd face i, I pick a number in the interval $(i, i+2)$ at random and if I get an even face j, I pick one at random from among the two numbers $\{j-1, j\}$. Let X be the number so selected. Find the distribution function F of X.

3. (a). Joint density of (X, Y) is given by

$$f(x, y) = \frac{e^{-y}}{y} \qquad 0 < x < y, \qquad 0 < y < \infty.$$

Calculate $E(X^2 \mid Y = 39)$.

(b). Show:

$$\sum_{k=n+1}^{\infty} e^{-n} \frac{n^k}{k!} \to \frac{1}{2}.$$

4. I have 100 numbered balls; half of them coloured green and half are red. These are placed in two boxes with 50 balls in each box. Everyday I pick one ball at random from **each** box and interchange: ball picked from box I is put in II and ball selected from II is put in box I. The state of the system on any day is the number of green balls in box I.

   (a). Compute transition matrix for the chain.
   (b). Make precise and show: In the long run it appears as if 50 out of the 100 balls are selected at random and placed in box I.

5. (a). In the usual one dimensional random walk, show: $\lim p_{0,50}^{(n)} = 0$.

   (b). I keep rolling a usual fair die independently. Let X_n be the face on n-th throw, for $n = 1, 2, \ldots$.
   Calculate $\lim_n P(\sum_1^n X_i$ is divisible by $10)$.

GOOD LUCK

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