Discrete Mathematics (Quiz 2)

1. Let G be a simple graph with n vertices and m edges. Show that if $m > \binom{n-1}{2}$, then G is connected. **10 points**

2. Show that the determinant of the Laplacian/Kirchoff matrix of a graph is zero. **10 points**

3. Let A be a graph obtained from the complete graph K_n by deleting an edge. What is the number of spanning trees of A? **10 points**

4. Given a connected graph $G = (V, E)$, let us define $M(G) = (E, I)$, where $I = \{F \subseteq E | (V, E - F) \text{ is connected}\}$. Prove that $M(G)$ is a matroid. **10 points**

5. Let G be a simple graph such that the minimum degree of any vertex is $\geq k$. If $k \geq 2$, show that there is a cycle in the graph of length at least $k + 1$. **10 points**

6. Let G be a regular bipartite graph (that is, a graph with all the vertices having the same degree). Prove that G has a perfect matching using Hall's theorem. **10 points**

$\gamma(v) \geq k \ \forall \ v \in G$

n^{n-2}, $\binom{n}{2}^{n}$

n^{n-1} $\binom{n-1}{2}$

G_1 G_2 v_2 v_3 v_4 v_i v_n v_1

let $\begin{bmatrix} n-2 & & -1 \\ n-1 & & \\ n-1 & & \\ -1 & & \\ & & \\ n-2 & n-1 \end{bmatrix}$

$= \det \begin{bmatrix} n-1 & -1 \\ & \\ n-1 \end{bmatrix} = \det$

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