Discrete Mathematics Assignment 2

Due Date: 10 PM, Feb 17

Write clear and concise solutions. It is fine to discuss with others, but your solutions must be in your own words that you have fully understood. Each question carries 10 marks.

1. Let w be any binary string of length k. For $n \geq k$, count the number of binary strings of length n that do not contain w as a *subsequence*.

2. Show that the n^{th} Catalan number $\frac{1}{n+1}\binom{2n}{n}$ counts the binary strings of length $2n$ that do not contain any of the following strings as *subsequences*: $1^{n+1}, 1^n0, 1^{n-1}0^2, \ldots, 1^i0^{n+1-i}, \ldots, 10^n, 0^{n+1}$.

3. Solve the following recurrence relation by relating it to a problem solved in class (or otherwise):

$$a_0 = 1$$
$$a_n = na_{n-1} + (-1)^n \text{ for } n \geq 1.$$

4. Give a combinatorial proof for the principle of inclusion exclusion

$$|\cap_{i=1}^n \overline{A_i}| = \sum_I (-1)^{|I|}|A_I|$$

by first moving all the negative terms in the summation to the LHS so that the equation assumes the form $A = B$, where both A and B are now sums of positive terms. Then define suitable sets whose sizes are these positive terms and give a bijective correspondence that will prove the equation.

5. Solve the recurrence relation $a_r + 3a_{r-1} + 2a_{r-2} = f(r)$ where $f(r) = 1$ for $r = 2$ and $f(r) = 0$ otherwise. Assume the boundary conditions $a_0 = a_1 = 0$.

6. Gossip is spread among r people via telephone. In a phone call between two people A and B, they exchange all the gossip they have heard so far. Let a_r denote the minimum number of phone calls so that all the gossip will be known to everyone. Show that $a_2 = 1$, $a_3 = 3$ and $a_4 = 4$. Then show that $a_r \leq a_{r-1}+2$. Finally, show that for $r \geq 4$, $a_r \leq 2r-4$.