

Discrete Mathematics Assignment 1

Due Date: Jan 23, 2023, 10.30 AM

Write clear and concise solutions. It is fine to discuss with others, but your solutions must be in your own words that you have fully understood.

1. Starting from $\mathbb{N} \times \mathbb{N}$ is countable, which we showed in class, show that the set of all rational numbers is countable.
2. If A_1, A_2, \dots, A_k are countable sets then show that $A_1 \times A_2 \times \dots \times A_k$ is countable.
3. Is the set of all *finite* subsets of \mathbb{N} countable? Justify your answer with proof.
4. What is the dimension of the reals, \mathbb{R} , as a vector space over the field of rationals, \mathbb{Q} ? Justify your answer with a proof.
5. Give an explicit injective map from $\mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$.
6. Give an explicit bijection from the open interval $(0, 1)$ to the reals \mathbb{R} .
7. What is the cardinality of the set of all continuous functions from $\mathbb{R} \rightarrow \mathbb{R}$? Justify with proof.
8. What is the cardinality of the set of all bijections from $\mathbb{N} \rightarrow \mathbb{N}$? Justify your answer with a proof.
9. Derive the well-ordering theorem assuming Zorn's lemma.
10. Derive the axiom of choice assuming Zorn's lemma.