

Diff Eqns Jan–Apr 2024 Quiz1 2024-02-08

- (1) Find (with suitable justification) the equation of the phase curve of the system  $\dot{x} = x, \dot{y} = 2y$ , passing through

(a) (8 marks)  $(x, y) = (1, 1)$ .

(b) (8 marks)  $(x, y) = (1, 0)$ .

In each case, draw a sketch of how the curve looks like in the  $x - y$ -plane.

- (2) Consider the system

$$\frac{dy}{dx} = \frac{y}{x}; x > 0, y > 0.$$

(a) (10 marks) Find the equation of the phase curves.

(b) (2 marks) Determine (with justification) whether there are equilibrium points.

- (3) (7 marks) Let  $k$  be a negative real number. Consider the system  $\dot{x} = kx, x > 0$  with initial condition  $(t_0, x_0)$ . Write down the solution to this system. Determine the point in  $t$  when the value of  $x$  is  $\frac{1}{4}x_0$ .

- (4) (10 marks) Let  $U$  be an open interval and  $v : U \rightarrow \mathbb{R}$  a  $C^1$  vector field. Let  $x_0 < x_1 \in U$  be such that  $v(x_0) = 0$  and  $v(\xi) \neq 0$  for all  $x_0 < \xi \leq x_1$ . Show that there is a constant  $c > 0$  such that  $|v(\xi)| \leq c|\xi - x_0|$ .

- (5) (15 marks) Consider the initial value problem  $\dot{x} = (1 - x)x$  with initial condition  $(t_0, x_0) = (0, 2)$ . Describe the equation of the integral curve. Determine (with justification) its vertical asymptote.