Classical Mechanics 1 at CMI: Final exam, Nov 25, 2022

This question paper has 2 pages. Read questions carefully, write your answers clearly in **blue** *and briefly explain your reasoning.* **Use a new sheet of paper where indicated.** *No computers/phones/notes/books/discussion permitted:* **closed book exam.** *90 marks.*

1. ⟨**10**⟩ Give an example of vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ in 3d Euclidean space to show that the cross product is not associative in general. Begin by giving an equation to say what it means for the cross product to be nonassociative.

2. ⟨**10**⟩ Give an example of a physical scattering process in which the total kinetic energy of particles in the far past is less than that in the asymptotic future. Explain briefly with a figure, mentioning the frame considered and the state of motion of the particles mentioned.

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3. ⟨**10**⟩ Suppose a potential in 3d Euclidean space is given by $V = \alpha/r^n$ where r is the radial distance from the origin, $\alpha > 0$ a real constant and n a real number.

   (a) ⟨**5**⟩ Find the associated force $\boldsymbol{F}$ and express it in spherical polar coordinates.

   (b) ⟨**2**⟩ For $n = -2$ write V and $\boldsymbol{F}$ in Cartesian coordinates.

   (c) ⟨**3**⟩ Describe the resulting $\boldsymbol{F}$ for $n = -2$ by saying whether the force is attractive/repulsive (relative to the origin), how it behaves with distance and give it a suitable name using standard terminology. Plot the graph of V.

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4. ⟨**10**⟩ Suppose a particle of mass m and charge q moves in $\mathbb{R}^3$ subject to the force $q\boldsymbol{v} \times \boldsymbol{B}(\boldsymbol{r})$ where $\boldsymbol{r}(t)$ is the position vector of the particle at time t and $\boldsymbol{v}(t)$ its velocity. $\boldsymbol{B}(\boldsymbol{r})$ is a **fixed** vector field called the magnetic field.

   (a) ⟨**2**⟩ Write Newton's equation of motion for this particle.

   (b) ⟨**5**⟩ Explain whether the motion of this particle is time-reversal invariant or not.

   (c) ⟨**3**⟩ Interpret in qualitative physical terms the equation that $\boldsymbol{r}(-t)$ satisfies.

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5. ⟨**10**⟩ Consider a plane simple pendulum with point-like bob of mass m suspended by a massless rod of length ℓ from a pivot subject to Earth's constant acceleration due to gravity g.

   (a) ⟨**3**⟩ Sketch a diagram of the phase space of the simple pendulum with counterclockwise angle of deflection θ. Indicate the origin and label axes.

   (b) ⟨**3**⟩ Indicate and label the two static solutions on the phase space mentioning their stability.

   (c) ⟨**4**⟩ Draw a librational and a rotational trajectory (with arrows showing increasing time) on the phase space and label which is which.

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6. ⟨10⟩ Equivalence principle.

   (a) ⟨7⟩ Give one experimental setup/thought experiment and related formulae & observations to motivate the principle of equivalence.

   (b) ⟨3⟩ State the principle of equivalence.

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7. ⟨10⟩ Kepler problem.

   (a) ⟨5⟩ What is the dimension of Kepler's constant K that arises in planetary physics? Obtain $[K]$ using an equation in which it appears, mentioning the meaning of symbols.

   (b) ⟨5⟩ Estimate the numerical value of K (as a ratio of real numbers) in suitable units using well-known quantities.

8. ⟨10⟩ Suppose S is an inertial frame with Cartesian coordinates x, y, z, t. Suppose frame S' moves along x at the speed of light c. Is S' an inertial frame? Explain your answer based on the special theory of relativity.

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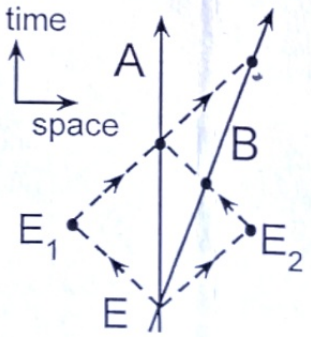
9. ⟨10⟩ In the context of special relativity, consider the accompanying space-time diagram showing the world lines of two inertial observers A and B in relative motion along with light signals to and from events E_1 and E_2. It is drawn in a frame in which A is at rest and in units where $c = 1$.

   (a) ⟨1⟩ What happens at event E?

   (b) ⟨2⟩ Infer whether the events E_1 and E_2 are simultaneous or which precedes the other for A and B (with brief justification).

   (c) ⟨2⟩ What general conclusion can one draw from the figure?

   (d) ⟨5⟩ Redraw the space-time diagram in a frame in which B is at rest.



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