I. Sc. Semester Program

Mid Semester Examination

Max. marks: 30 Duration: 120 minutes

Answer all questions. The marks for each question is indicated at the end of the question. Your answer should be legible, complete and logically correct in order to get full marks. Please write the precise statements of the results that you use in your solutions.

1. State and prove Goursat's theorem for a rectangular contour. [6]

2. Let $F(z) = \sum a_n (z - z_0)^n$ be a power series with +ve radius of convergence R. Then show that F is holomorphic in $D(z_0, R)$ and the derivative F' is given by another power series that has same radius of convergence R as F. [6]

3. For which values of real constants a, b, c, d is the function $u(x,y) = ax^3 + bx^2y + cxy^2 + dy^3$ harmonic? Determine a harmonic conjugate of u in the cases where it is harmonic. [6]

4. Find all the roots of the equation $\cos z = 2$. [6]

5. Evaluate the following integral where C denote the +vely oriented bdry of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. [6]

$$\int_C \frac{e^{-z}}{z - (\pi i/2)} dz$$