B.Sc. Complex Analysis

End Semester Examination

Max marks: 35 Duration: 120 min

Answer all the questions. All questions carry equal marks. Your answers should be legible, logical and complete in order to gain full points. Quote the precise statement of the results that you used in your solutions.

1. Spse $f : G \to \mathbb{C}$ be a holomorphic function where G is a domain in $\mathbb{C}$. Show that the set of zeros of f ~~are~~ is isolated.

2. Let $G \subset_o \mathbb{C}$ and $f : G \to \mathbb{C}$ be a map. When do you say that f is conformal? Show that if f is conformal then f is holomorphic on G.

3. Let C denote the line segment from $z = i$ to $z = 1$. Without evaluating the integral, show that $\left| \int_C \frac{dz}{z^4} \right| \le 4\sqrt{2}$.

4 (a) Let $f(z) = \dfrac{\mathrm{Log}(z+4)}{z^2 + i}$ where Log denotes the principal branch of logarithm. [3]
Find the domain on which f is analytic.

(4)(b) Let $C_0 : |z - z_0| = R$ taken in +ve orientation. S.T.

(i) $\displaystyle\int_{C_0} \frac{dz}{z - z_0} = 2\pi i$ (ii) $\displaystyle\int_{C_0} (z - z_0)^{n-1}\, dz = 0$, $n \in \mathbb{Z} \setminus \{0\}$. [4]

5. Let $f = u + iv$ be analytic in a domain D and consider for $c_1, c_2 \in \mathbb{R}$, $u(x,y) = c_1$, $v(x,y) = c_2$. S.T. if $z_0 = (x_0, y_0)$ is a pt in D which is common to two particular curves $u(x,y) = c_1$ and $v(x,y) = c_2$ and if $f'(z_0) \neq 0$, then the lines tangent to those curves at (x_0, y_0) are $\perp^r$ to each other.