Calculus
Quiz 6
06/11/'23

You may use your class notes during the quiz. No other sources are permitted

Name: ____

1. Let $e_1, \dots, e_n$ be the standard basis of $\mathbb{R}^n$, and let $\phi_1, \dots, \phi_n$ be the dual basis.

   (a) Show that $\phi_{i_1} \wedge \cdots \wedge \phi_{i_k}(e_{i_1}, \dots, e_{i_k}) = 1$.

   (b) Let $v_1, \dots, v_k \in \mathbb{R}^n$, and let A be the $k \times n$ matrix $(v_{i,j})$ whose (i,j)-th entries are the coefficients in the expressions $v_i = \sum_j v_{i,j} e_j$, $i, j = 1, \dots, n$. Show that $\phi_{i_1} \wedge \cdots \wedge \phi_{i_k}(e_1, \dots, e_k)$ is the determinant of the $k \times k$ minor of A obtained by selecting columns $i_1, \dots, i_k$.

   (c) Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a linear map. Let $f^* : \wedge^k \mathbb{R}^n \to \wedge^k \mathbb{R}^n$ be the pullback of k-forms. When is f^* injective?