Calculus
Quiz 4
27/09/'23

You may use your class notes during the quiz. No other sources are permitted

Name: _____

1. Consider the function f defined on the unit square $Q : [0,1] \times [0,1] \subset \mathbb{R}^2$ as follows: $f(x,y) = 1$, if $x \in \mathbb{Q}$, and $f(x,y) = 2y$, if $x \notin \mathbb{Q}$.

   (a) Show that $\int_0^t f(x,y)dy$ exists for $t \in [0,1]$ and that

   $$\underline{\int_0^1} \left[\int_0^t f(x,y)dy \right] dx = t^2, \text{ and } \overline{\int_0^1} \left[\int_0^t f(x,y)dy \right] dx = t.$$

   Thus, $\int_0^1 [\int_0^1 f(x,y)dy]dx$ exists and equals 1.

   (b) Show that $\int_0^1 [\overline{\int_0^1} f(x,y)dx]dy$ exists. Find its value.

   (c) Prove that the integral $\int_Q f$ does not exist.