Calculus
Quiz 2
30/08/'23

You may use your class notes during the quiz. No other sources are permitted

Name: _____

1. Let A and B be rectangles, $C \subset A \times B$ a set of content 0.

   (a) For $x \in A$, let $B_x := \{y \in B : (x, y) \in C\}$. Define a subset $A' \subset A$ as

   $$A' := \{x \in A : B_x \text{ is not of content } 0\}.$$

   Show that A' has measure 0. (Hint: Fubini)

   (b) Take $A = B = [0, 1]$, and let $C \subset A \times B$ be the set

   $$\bigcup \left(\{p/q\} \times [0, 1/q]\right),$$

   where the union is over all $p/q \in \mathbb{Q} \cap [0, 1]$ with $(p, q) = 1$. Use C to show that the word "measure" in the previous part cannot be replaced with "content."