Calculus I
Quiz 1
15:30-16:00, 22 August, 2019

You may use your class notes during this quiz, but no other source is permitted.

1. If $P(x,y) = xe^{-y^2}$ and $Q(x,y) = -x^2ye^{-y^2} + 1/(x^2+y^2)$, evaluate the line integral $\oint_C P dx + Q dy$ around the boundary of the square determined by the inequalities $|x| \leq a$, $|y| \leq a$.

2. Let R be the closed unit disc $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$, and S an open set containing R. Let $u, v \in \mathcal{C}^1(S)$, and define vector fields $\vec{f}$ and $\vec{g}$ as follows:

$$\vec{f}(x, y) = v(x, y)\vec{i} + u(x, y)\vec{j}, \qquad \vec{g}(x, y) = \left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{i} + \left(\frac{\partial v}{\partial x} - \frac{\partial v}{\partial y}\right)\vec{j}$$

Compute $\iint_R (\vec{f} \cdot \vec{g}) dx dy$ if $u(x, y) = 1$ and $v(x, y) = y$ on the boundary of R.