Analysis Quiz 3

September 2023

Instructions

1. Each question carries 10 marks.

2. Maximum marks is 30.

3. Crying is allowed but please do it silently. Do not disturb others.

Question 1

1. Let X be a compact metric space and $\{K_n\}$ is a collection of closed subspaces such that $K_{n+1} \subseteq K_n$. Show that $\bigcap K_n \neq \phi$. $K_n \neq \phi$

2. Does the result hold if X is not compact. Give an example.

Question 2

1. Let $f : [0,1] \to Y$ is a continuous bijective function. Prove that f^{-1} is continuous.

2. Does the result hold if we replace $[0,1]$ with $[0,1)$.

Question 3

For a compact metric space (X,d) show that $\exists\ u,v$ s.t. $d(u,v) = \operatorname{diam} X$.
[Diam $X = \sup \{d(x,y)|\ x,y \in X\}$]

Bonus question 1

Let f be a continuous real valued function on $\mathbb{R}^n$ such that $|f(x)| \geq c \cdot ||x||$. Show that f is proper.
[f is called proper if preimage of any compact set is compact]

Bonus question 2

Let (X,d) be a compact metric space. Let $f : X \to X$ be a function such that $d(f(x), f(y)) = d(x,y)$. Show that f is bijective.