Analysis 2 - Quiz 2

Duration: 1 hour

Maximum Marks: $4 \times 10 = 40$

1. Let A and B be dense subsets of a metric space X.
   (a) Is $A \cap B$ necessarily dense?
   (b) What if in addition A and B are open?

2. Let (X, d) be a metric space and $f\colon X \to \mathbb{R}$ a continuous function. Define the graph of f as

$$\Gamma := \{(x, f(x))\colon x \in X\} \subseteq X \times \mathbb{R}.$$

   Show that Γ is closed in $X \times \mathbb{R}$.

3. Define $f_n\colon (0,1) \to \mathbb{R}$ by $f_n(x) = \frac{1}{nx+1}$. Show that the sequence $\{f_n(x)\}_{n \geq 1}$ converges pointwise but not uniformly over $(0,1)$.

4. Let $f_n\colon [0,1] \to \mathbb{R}$, $n \geq 1$, be a sequence of functions converging uniformly to $f\colon [0,1] \to \mathbb{R}$. Prove or disprove: f has finitely many discontinuities if f_n has finitely many discontinuities for each n.

$\frac{1}{2n+1} < \cancel{x} < \frac{1}{n+1}$

$(0,1)$

$\mathbb{Q} = \{r_1, r_2, \dots\}$ ~~...~~ $\frac{1}{i}$

r_i

$i \geq n$