ANALYSIS I, 2022, Quiz 1
Friday, September, 2022
Duration: 30 minutes

1. (1 point) Let $\mathbb{C}$ be the field of complex numbers. Show that no order can be defined on $\mathbb{C}$ that makes it an ordered field.

2. (6 points) Determine if the following sets of real numbers are bounded above or bounded below and where applicable, find their supremum or infimum.

   $\mathbb{N} = \{1, 2, \ldots\}$ denotes the set of positive natural numbers.

   (a) $S_1 = \left\{ \frac{3k+4}{k+2} \mid k \in \mathbb{N} \right\}$.

   (b) $S_2 = \left\{ \frac{1}{a} - \frac{1}{b} \mid a, b \in \mathbb{N} \right\}$.

3. (3 points) Le S be a bounded set of real numbers (i.e., it is bounded above and bounded below). Let M, m denote the supremum and infimum of S, respectively. Consider the set

$$A = \{ |x - y| \mid x, y \in S \}.$$

   Is it true that A is bounded above and its supremum is $M - m$?