Algebra 1 Quiz 1 Your roll number: ....

Throughout the quiz $r = 100 +$ the last two digits of your roll number, e.g., $r = 175$ for BMC202275. $e_1, e_2, \ldots$ denote the standard basis vectors of $\mathbb{R}^n$.

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1. (a) Fill in the blank with a word/short phrase: the two vectors e_1 and $e_1 + re_2$ form a basis of $\mathbb{R}^2$ because neither of the given vectors is a ......of the other, but the same reasoning is not enough to justify that a given set of three vectors is a basis of $\mathbb{R}^3$.
Write your answer here: ____________________

(b) Find the coordinates of $2022e_1 + e_2$ with respect to the basis in (a).

2. Consider the given conditions on some $r \times c$ matrix A and the associated function $f(x) = Ax$, where x is a column vector of the appropriate size.

(I) The equation $Ax = b$ in unknown x has a solution for every possible column vector b, i.e., the function f is surjective (onto).

(II) The equation $Ax = 0$ in unknown x has no solution other than $x = 0$, which is equivalent to saying that the function f is injective (one-to-one). Injectivity of f clearly implies the earlier condition. The earlier condition implies injectivity because, if $f(x) = f(y)$, i.e., if $Ax = Ay$ then ......, which forces $x = y$ by the given condition. Fill in the blank here with something very short:

For each of the statements below, state next to it whether it implies/is implied by/is equivalent to (I) or none of these, AND whether it implies/is implied by/is equivalent to (II) or none of these.

Your answer will consist of two parts: One of the following: $\Rightarrow I$ $\Leftarrow I$ $\Leftrightarrow I$ NONE

followed by one of the following: $\Rightarrow II$ $\Leftarrow II$ $\Leftrightarrow II$ NONE

Rows of A are linearly independent

Rows of A span $\mathbb{R}^c$

Columns of A are linearly independent

Columns of A span $\mathbb{R}^r$

Each row of an echelon form of A has a pivot

Each column of an echelon form of A has a pivot

$r \leq c$

$c \leq r$

3. A 4×5 matrix M has column vectors v_1, v_2, v_3, v_4, v_5. Suppose M is row equivalent to the matrix given on the board. Write (a) a basis for the row space of M, (b) a basis for the column space of M, (c) the dimension of the null space of M and then a basis for this space. **Clearly label your answer to each individual part.** Consider carefully whether/how much row reduction is necessary to answer the question.

4. (a)Let E_{ij} denote an $n \times n$ matrix with the sole nonzero entry being 1 in row i and column j. Write the product $E_{ij}\, E_{kl}$.

(b) For $n = 2$ find all matrices M such that $E_{12}M = ME_{12}$.

(c) In general, for a fixed square matrix A, how will you find all matrices M such that $AM = MA$? Do not actually try to do any calculation, just describe the procedure you will carry out. Does it make sense to count such matrices using the concept of dimension?

5. For each of the given matrices, find its inverse or state why it is not invertible.