ACT I: Assignment I

Due date: 14th Spetember 2024

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1. All the statements proven in the class or given in the practice problems can be used without proof. Other than that, anything you use needs to be proven.
2. Encouraged, but not compulsory, to write the solutions in Latex.
3. Allowed to discuss with others but write the solutions independently.

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1. (**10 marks**) Let C be an $[n,k,d]_q$ code over a finite field $\mathbb{F}_q$ with the generator matrix G. If G does not have a column containing all zeros, then show that
$$\sum_{\mathbf{c}\in C} \mathrm{wt}(\mathbf{c}) = n(q-1)q^{k-1},$$
where $\mathrm{wt}(\mathbf{c})$ denotes the number of nonzero coordinates in $\mathbf{c} \in \mathbb{F}_q^n$.

2. (**20 marks**) Let C be an $[n,k]_q$ code with the block length and the dimension of C are n and k, respectively. The code C is called *self-dual* if $C = C^{\perp}$, that is, the code C is the same as its dual. For any prime q, is there an $[8,4]_q$ self-dual code over $\mathbb{F}_q$?

3. (**10 marks**)The set of all $n_2 \times n_1$ matrices over $\mathbb{F}_2$ forms a vector space V of dimension $n_1 n_2$. For $i = 1,2$, let C_i be an $[n_i, k_i, d_i]_2$ linear code over $\mathbb{F}_2$. Let C be the subsets of V consisting of those matrices for which every column, respectively every row, is a codeword in C_1, respectively C_2. Show that C is an $[n_1 n_2, k_1 k_2, d_1 d_2]_2$ code. The code C is called *direct product* of C_1 and C_2.

4. (**10 marks**) Show that $[15,8,5]_2$ code does not exist.

5. Show the following.
   (a) (**10 marks**) If there exists an $[n,k,d]_q$ code, then there exists an $[n-d, k-1, d']_q$ code with $d' \geq \lceil d/q \rceil$.
   (b) (**5 marks**) For an $[n,k,d]_q$ code,
   $$n \geq \sum_{i=0}^{k-1} \lceil d/q^i \rceil.$$
   It is known as *Griesmer Bound*.

6. (**15 marks**) Let $q \geq 2$ be an integer. Let $\delta \in (0, 1-\frac{1}{q})$. Let $\epsilon \in [0, 1-H_q(\delta)]$ and n be a positive integer. Let $k = (1-H_q(\delta)-\epsilon)n$. Let H be an $(n-k)\times n$ matrix over $\mathbb{F}_q$ picked uniformly and randomly. Then, show that H is a *parity* matrix of a code of block length n, rate $1-H_q(\delta)-\epsilon$ and relative distance at least δ with probability $\geq 1-q^{-\epsilon n}$.